Class XI Mathematics
Permutation: Theory

- **Factorial Notation:** Let n be a positive integer. Then the continued product of first n natural numbers is called factorial n, to be denoted as $n!$ or $\lfloor n$

  Thus $n! = n(n-1)(n-2)...3.2.1$

  When n is a negative or a fraction, $n!$ is not defined

  Also AS A SPECIAL CASE WE DEFINE $0! = 1$

  Example: $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

  $7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$

- **Multiplication Principle**: If an event can occur in m different ways and if following it, a second event can occur in n different ways, then the two events can occur in succession in $m \times n$ ways.

  Example: If we can arrange 6 boys in a row in 6! ways and 5 girls in 5! Ways, then total number of ways of arranging both is 6! X 5! Ways.

  Similarly for events occurring in m, n, p,… different ways, we have the total number of ways in which they can occur in the stated order as $(m \times n \times p \times ...)$ ways

  Q) Vanisha wants to make a secret 3 letter code using 26 alphabets of English. In how many ways can she make the code if

  a) Alphabets are allowed to be repeated

  b) Alphabets are not allowed to be repeated

- **Addition Principle**: If two events can occur independently in exactly m and n ways respectively, then either of the two events can occur in $(m+n)$ ways

  Q) How many 2 digit even numbers can be formed from the digits 1, 2, 3, 4, 5 if the digits can be repeated?

  Q) Find the number of different signals that can be generated by arranging at least 2 flags in order (one below the other) on a vertical staff, if five different flags are available.

  Q) Find the number of 4 letter words, with or without meaning, which can be formed out of the letters of the word ROSE, where the repetition of the letters is not allowed.

- **Permutation:** The different **ARRANGEMENT** that can be made out of a given number of things taking all or some at a time are called Permutation

$$P(n,r)=\frac{n!}{(n-r)!}$$

- The number of **Permutations** of n different things, taken all at a time is given by $P(n,n)=\frac{n!}{(n-n)!}=n!$
- The number of all permutations of n different things taken r at a time, when a particular thing is to be included in each arrangement is

$$r.P(n-1,r-1)$$

- The number of all permutations of n different things taken r at a time, when a particular thing is never included in each arrangement is $P(n-1,r)$
- If there are n objects of p_1 are alike of one kind, p_2 are alike of 2nd kind …..p_r are alike of the rth kind such that $p_1+p_2+\ldots+p_r=n$; then the number of permutations of these n objects is $\frac{n!}{(p_1)!\times(p_2)!\times\ldots\times(p_r)!}$

For example:

If we have to find the number of permutations of the letters of the word ALLAHABAD, we find that there are 9 objects (letters) of which there are 4A's, 2 L's and rest are all different.

Therefore, the required number of arrangements $\frac{9!}{4!\times 2!}=\frac{5\times6\times7\times8\times9}{2}=7560$

Q1: In how many ways can 4 red, 3 yellow and 2 green discs be arranged in a row if the discs of the same colour are indistinguishable?

Ans: $\frac{9!}{4!\times3!\times2!}=1260$

- The number of permutation of n different things taken r at a time when each may be repeated any number of times in each arrangement is n^r
- The number of circular permutations of n different objects is $(n-1)!$
- The number of ways in which beads of a necklace can be arranged is $\frac{1}{2}(n-1)!$