Class 11 Physics

Motion in a Plane – Chapter Summary

1. Scalars and vectors. Complete idea of position vectors and displacement vectors

   The position vector of an object in the x-y-z plane is given by $r = x\hat{i} + y\hat{j} + z\hat{k}$

   Displacement vector:

   $\Delta r = r_2 - r_1 = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

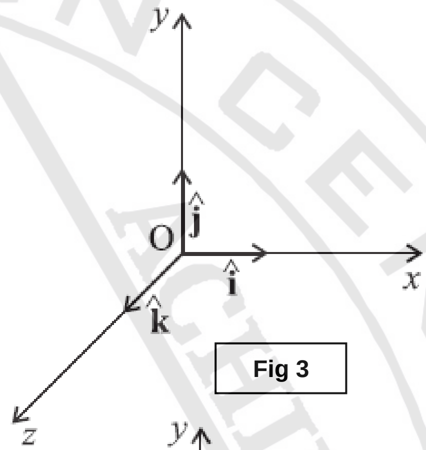

Fig 3

2. Any Vector A may be represented as $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$, where A_x, A_y and A_z are the components of A along x, y and z axes

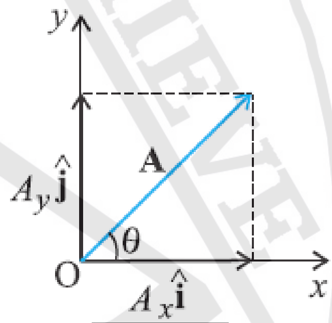

Fig 4

3. **Unit Vector**: A unit vector associated with a vector **A** has magnitude one and is along the vector **A**. It is given by $\hat{A} = \frac{\vec{A}}{|A|}$, where $|A|$ is the magnitude of $\vec{A}$ and is given by $|A| = \sqrt{A_x^2 + A_y^2 + A_z^2}$ Unit vectors along the *x*-, *y* and *z*-axes of a rectangular coordinate system are denoted by $\hat{i}, \hat{j}$ and $\hat{k}$

   If we multiply a unit vector, say **n** by a scalar, the result is a vector $\lambda = n\lambda$. In general, a vector **A** can be written as **A** = | **A**| **n** where **n** is a unit vector along **A**.

4. **Resolution of vectors:** A vector A may be resolved into components A_x and A_y along x and y axis as shown in fig 4

   $A_x = A\cos\theta, A_y = A\sin\theta$ and $A = \sqrt{A_x^2 + A_y^2}$

5. **Triangle law of vector addition**. Statement and derivation of the formulas:

$R=\sqrt{A^2+B^2+2AB\cos\theta}$ and $\tan\alpha=\dfrac{B\sin\theta}{A+B\cos\theta}$, Where R is the resultant of two vectors A and B. θ is the angle which A makes with the horizontal and α is the angle between R and A

6. **Parallelogram law of vector addition**. Statement and derivation of the formulas:

   $R=\sqrt{A^2+B^2+2AB\cos\theta}$ and $\tan\alpha=\dfrac{B\sin\theta}{A+B\cos\theta}$, Where R is the resultant of two vectors A and B. θ is the angle between the vectors and α is the angle between R and A

7. Acceleration is defined as the rate of change of velocity. $\bar{a}_{av}=\dfrac{v_2-v_1}{t_2-t_1}=\dfrac{\Delta v}{\Delta t}$ where the velocity of an object at two different times t_1 and t_2 are v_1 and v_2

8. The **acceleration** (instantaneous acceleration) is the limiting value of the average acceleration as the time interval approaches zero :

   $$a=\lim_{\Delta t\to 0}\frac{\Delta v}{\Delta t}=\frac{dv}{dt}$$

   If as shown in point 3 of motion in a straight line if $x=6t^2-5t+9$ is the position of an object, then average acceleration is found as follows:

   First we differentiate x w.r.t to get v $v=\dfrac{dx}{dt}=12t-5$. Next we find v at the two given times say t_1 = 2s and t_2 = 6s $v_1=12(2)-5=19ms^{-1}$ and $v_2=12(6)-5=67ms^{-1}$

   $$\Rightarrow a_{av}=\frac{v_2-v_1}{t_2-t_1}=\frac{67-19}{6-2}=\frac{48}{4}=12ms^{-2}$$

   To obtain **acceleration** (instantaneous acceleration), $a=\dfrac{dv}{dt}=\dfrac{d(12t-5)}{dt}=12(1.t^{1-1})-0=12ms^{-2}$ Note that in this case the acceleration (instantaneous acceleration) does not depend on time and is a constant.

9. **Projectile Motion (**Most Important) An object that is in flight after being thrown or projected is called a **projectile**. Such a projectile might be a football, a cricket

ball, a baseball or any other object. The motion of a projectile may be thought of as the result of two separate, simultaneously occurring components of motions. One component is along a horizontal direction without any acceleration and the other along the vertical direction with constant acceleration due to the force of gravity.

Suppose that the projectile is launched with velocity u that makes an angle θ with the *x*-axis as shown in Fig. After the object has been projected, the acceleration acting on it is that due to gravity which is directed vertically downward:

$a = -g\hat{j};\ \therefore a_x = 0 \text{ and } a_y = -g$;

The components of initial velocity u are : $u_x = u\cos\theta \text{ and } u_y = u\sin\theta$

a. Equation of a projectile:

$y = x(\tan\theta) - \frac{g}{2(u\cos\theta)^2}x^2$ Since θ and u are constants this equation is of the form $y = ax + bx^2$, in which *a* and *b* are constants. This is the equation of a parabola, i.e. the path of the projectile is a parabola

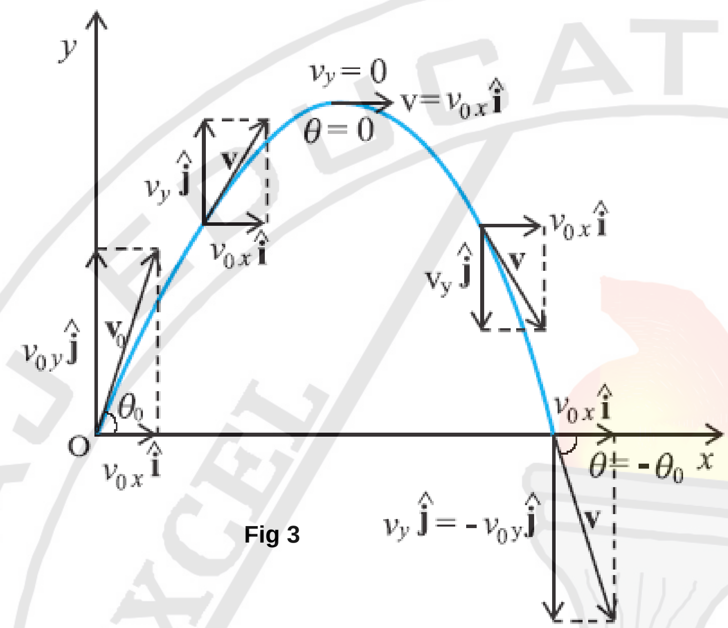

Fig 3

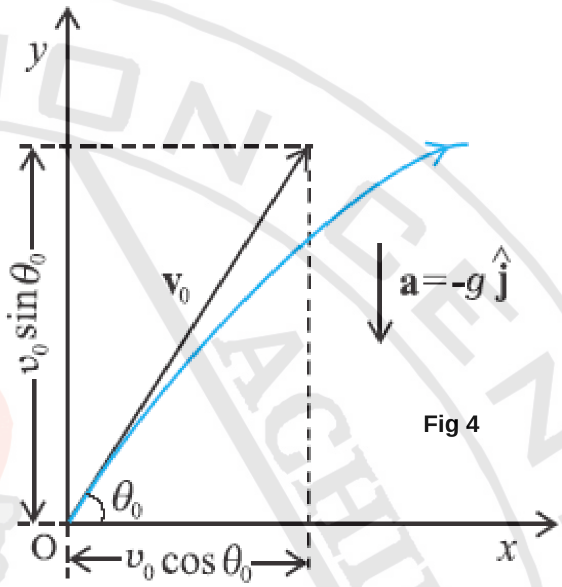

Fig 4

b. Time of flight : $T = \dfrac{2u\sin\theta}{g}$

c. Range: The horizontal distance travelled by a projectile from its initial position (x = y = 0) to the position where it passes y = 0 during its fall is called the **horizontal range**, R. It is the distance travelled during the time of flight T Therefore, the range R is $R = \dfrac{u^2 \sin 2\theta}{g}$

d. Maximum height: $h = \dfrac{(u\sin\theta)^2}{2g} = \dfrac{u^2\sin^2\theta}{2g}$

e. Velocity at an instant The components of velocity at time t are

$$v_x = u_x + a_x t \Rightarrow v_x = u\cos\theta + (0)t = u\cos\theta$$

$$v_y = u_y + a_y t \Rightarrow v_x = u\sin\theta + (-g)t = u\sin\theta - gt \quad \therefore v = \sqrt{{v_x}^2 + {v_y}^2}$$

Also as is clear form fig. 4, $\tan\theta = \dfrac{v_y}{v_x} \Rightarrow \theta = \tan^{-1}\left(\dfrac{v_y}{v_x}\right)$

10. Uniform Circular motion: When an object follows a circular path at a constant speed, the motion of the object is called **uniform circular motion**. The word

"uniform" refers to the speed, which is uniform (constant) throughout the motion. Since the velocity of the object is changing continuously in direction, the object undergoes acceleration. This acceleration of an object moving with speed v in a circle of radius R has a magnitude v^2/R and is always **directed towards the centre**. This is why this acceleration is called **centripetal acceleration** (a term proposed by Newton)

"Centripetal" comes from a Greek term which means 'centre-seeking'. Since v and R are constant, the magnitude of the centripetal acceleration is also constant. However, the direction changes — pointing always towards the centre. Therefore, a centripetal acceleration is not a constant vector.

Derivation of the relation: $a_c = \frac{v^2}{R} = \frac{\omega^2 R^2}{R} = \omega^2 R$ ➔**Important**

Additionally you should know

a. **Dot product of vectors**: $\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta,$ where θ is the angle between the vectors

b. **Cross product of vectors** $\vec{A}\times\vec{B} = |\vec{A}||\vec{B}|\sin\theta,$ where θ is the angle between the vectors